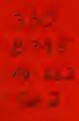


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Faculty Working Papers

PRICING OF LIQUIDITY FOR PREFERRED STOCKS ON THE NEW YORK STOCK EXCHANGE

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#662

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Faculty Working Papers

INTERACTION BETWEEN MONETARY AND FISCAL POLICIES

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#664

College of Commerce and Business Administration
University of Illinois at Urbana-Champaign



FACULTY WORKING PAPERS

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University of Illinois at Urbana-Champaign

March 28, 1980

INTERACTION BETWEEN MONETARY AND FISCAL POLICIES

William R. Bryan, Professor, Department of Finance A. James Heins, Professor, Department of Economics Morgan J. Lynge, Jr., Assistant Professor, Department of Finance

#664

Summary

The purpose of this paper is to explore the existence of interactions between monetary and fiscal policy in their combined impact on economic activity. Two models using dummy variables to capture different states of policy are estimated. Results using a model in which policy effects are additive differ little from results in which interaction among policies is allowed. In addition to detecting no significant interaction effects it is observed that monetary policy has a greater impact on GNP than does fiscal policy. This result is generated regardless of the model used.

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INTERACTION BETWEEN MONETARY AND FISCAL POLICIES

by William R. Bryan, A. James Heins, and Morgan J. Lynge, Jr.*

The purpose of this paper is to explore interactions between monetary and fiscal policy in their combined impact on economic activity.

The question is: "do models that allow money and government deficits to interact in the process of income determination have more explanatory power than models in which their effects are additive?" In the process of finding the "no" answer to that question we turn up additional evidence that monetary actions are indeed more important than fiscal actions.

These questions and answers stem from the work of Andersen and Jordan (1968) and the flow of comment stimulated by that work. Much of this commentary focused on the inadequacy of reduced-form models and the choice of monetary and fiscal variables. Here we turn to another dirension of the efficacy of monetary and fiscal policies—interaction between these policies. 1

This note proceeds as follows: First, we lay the groundwork for a meaningful discussion of "interaction." Second, we discuss the models to be estimated. Next, the statistical results are presented. Finally, we summarize our findings.

Interaction

There is a widely-held view that a given rate of increase in aggregate demand can emerge from many alternative combinations of monetary and fiscal policies. For illustrative purposes consider Figure 1. The vertical axis measures rates of change in aggregate

demand; the horizontal axis measures "expansive" or "restrictive" fiscal policy; and the third dimension measures the "ease" or "tightness" of monetary policy.

Figure 1 is drawn as linear in both its variables and its arguments—a plane could rest on the points depicted by the "flagpoles". Aggregate demand is not affected by an interaction between monetary policy. Monetary policy within the context of a restrictive fiscal policy would have the same effect on aggregate demand as would be the case if fiscal policy were expansive. The work of Andersen-Jordan and of their detractors is essentially of this character, with no effort to account for interaction between monetary and fiscal policies.

However, there is reason to believe that the impact of a fiscal policy change depends upon the monetary-policy environment within which it occurs. Similarly, the impact of monetary policy depends upon the fiscal policy environment. Thinking within the conventional IS-LM context, changes in income resulting from a shift in fiscal policy would depend upon the shape of, and shifts in, the LM schedule; also, the impact of monetary policy would depend upon the shape of, and shifts in, the IS schedule. Specifically, there is a presumption that the appropriate surface for Figure 1 would be generally concave to the origin—though it need not be well—behaved.

What is required is a model that permits nonlinearities to be freely estimated. It would seem that we need a model that would faithfully replicate whatever surface would be appropriate for Figure 1—assuming that the plane shown there is inadequate.

The Model

The clue leading to construction of a useful statistical model emerged from an examination of Figure 1. Instead of attempting to model the "true" surface, it occurred to us that it would be possible to estimate the height of the supporting "flagpoles." This could be accomplished by an appropriate arrangement of "dummy" variables.

First, both the monetary and fiscal policy variables could be scaled. For illustrative purposes, suppose the variables were scaled to conform to the number of "flagpoles" in Figure 1—that is, in terms of "restriction" or "expansion" in the policy instruments (setting aside questions of measurement). Such a classification system would create 9 cells, corresponding to the potential policy combinations (see Table I).

Table I
ALTERNATIVE POLICY COMBINATIONS

	Fiscal Policy:			
Monetary Policy:	Expansive (FPE)	Neutral (FPN)	Restrictive (FPR)	
Expansive (MPE)	x_1	x_2	x ₃	
Neutral (MPN)	x_4	x ₅	^x 6	
Restrictive (MPR)	x ₇	x8	x ₉	

The usual approach for using dummy variables in such an application would be to create three 0 or 1 values for the alternative monetary policy variables (actually one variable must be dropped) and three 0 or 1 values for the alternative fiscal policy variables. This done, the following regression model would be estimated:

(1)
$$y = a_0 + a_1 \text{ MPE} + a_2 \text{ MPR} + a_3 \text{ FPE} + a_4 \text{ FPR} + E_1;$$

where, in addition to symbols previously defined, y = aggregate demand.

Suppose the following estimates were obtained: $\hat{a}_0 = .02$, $\hat{a}_1 = .08$, $\hat{a}_2 = -.03$, $\hat{a}_3 = .05$, and $\hat{a}_4 = -.04$. The estimates of aggregate demand that could emerge from this model are summarized in Table II.

Table II
POLICY COMBINATIONS AND AGGREGATE DEMAND

	Fiscal Policy			
	Expansive (FPE)	Neutral (FPN)	Restrictive (FPR)	
	%	%	%	
Expansive (MPE)	15	10	6	
Neutral (MPN)	7	2	-2	
Restrictive (MPR)	4	-1	- 5	

In this model, the effects of monetary and fiscal policy are additive. The only thing accomplished by the dummy variable approach is to permit a non-linearity to emerge in each variable considered separately. As fiscal policy moves from expansive to netural there is a five percentage point decline in aggregate demand; however, the movement from neutral to restrictive involves a decline of four percentage points. Similarly, as monetary policy shifts from expansive to neutral aggregate demand declines by eight percentage points; a further shift to restrictive results in only a three percentage point decline in aggregate demand. Note, however, that the effects of changes in, say, monetary policy do not depend upon the status of fiscal policy. There is no interaction.

A different dummy-variable approach would be to estimate each "flag-pole" separately--with a dummy for each policy combination. The model to be estimated is $(X_1$ through X_2 are identified in Table I):

(2)
$$y = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_4 + b_4 x_5 + b_5 x_6 + b_6 x_7 + b_7 x_8 + b_8 x_9 + E_2$$

The estimated values of the dependent variable corresponding to each policy mode would be calculated from the estimates of the b's.

If in fact there were no interaction the b coefficients of equation

(2) could be calculated from the a coefficients of equation (1) as follows:

$$b_0 = a_0 + a_1 + a_4$$
 $b_1 = a_3 - a_4$
 $b_2 = -a_4$
 $b_3 = a_3 - a_1 - a_4$
 $b_4 = -a_1 - a_4$
 $b_5 = -a_1$
 $b_6 = a_2 + a_3 - a_1 - a_4$
 $b_7 = a_2 - a_1 - a_4$
 $b_8 = a_2 - a_1$
 $b_4 = -a_1 - a_4$

That is, the estimates of the dependent variable obtained from (2) would not differ from those obtained from (1). However, if there were interaction—that is, if the impact of monetary policy depends upon the status of fiscal policy, and conversely—the estimates of b_0 through b_8 would not be derivable from estimates of a_0 through a_4 . To that empirical question we now turn.

Data and Results

The raw data consist of quarterly observations on GNP, the seasonally adjusted money supply (M1), and the high employment deficit or surplus.²

The observations span the period from the first quarter of 1952 through the fourth quarter of 1978.

Consistent with the models cutlined above, the dependent variable (GNF) was converted to quarter-over-quarter percentage changes. The various states of policy are represented by dummy variables. The monetary policy dummies were based on the quarter-over-quarter percent change in seasonally adjusted M1. The 108 quarterly percent changes were ranked, and the 36 highest growth rates were designated "expansive," the next 36 growth rates were designated "neutral," and lowest 36 growth rates were "restrictive." The boundaries of these three categories and the monetary policy dummy values are shown in the top panel of Table III. The fiscal policy dummies were based on the change in the high employment deficit or surplus for each quarter. The 36 largest negative changes are "expansive" and so on. The lower panel of Table III gives the boundaries and values for the fiscal policy dummies.

_ Table III
DEFINITION OF POLICY DUMMY VALUES

Range of values for % Δ in M1	MPE	MPN	MPR	Value of Median Observation
1.295 to 2.484	1	0	0	1.76
0.717 to 1.287	0	1	0	0.99
-0.623 to 0.705	0	0	1	0.28
Range of values for Δ HED (\$ billions)	FPE	FPN	FPR	
-48.1 to -1.2	1	0	0	-3.2
-1.0 to +1.3	0	1	0	-0.3
1.4 to 26.8	0	0	1	-2.7

The dummy variables attempt to account for the impact of monetary and fiscal policies on GNP growth. To improve the specification of the proposed models, the impact of changes in private demand that are not induced by economic policies should be included in an explanation of GNP growth. This private demand effect is captured in the variable PD. In order to isolate the effects of private demand on GNP growth that are independent of current monetary and fiscal policies, the following technique was used. Gross domestic investment (GDI) was chosen as a measure of private demand. The quarterly percent change in CDI was regressed on the quarterly percent change in M1 and the change in the high employment deficit. The residuals from this regression contain that portion of CDI that is not explained by monetary and fiscal policies. This residual was then used as the variable PD.

Before discussing the test results, it is of interest to inspect the distribution of combinations of monetary and fiscal policies. Table IV shows the number of observations occurring for each policy combination. The numbers in in parentheses in Table IV represent the theoretical distribution of observations that would emerge if there were independence between monetary and fiscal policy. The actual and theoretical distributions are not significantly different.

Table IV
DISTRIBUTION OF POLICY COMBINATIONS

Monetary	Fiscal Policy			
Policy	Expansive (FPE)	Neutral (FPN)	Restrictive (FPR)	
Expansive (MPE)	11	9	16	
	(12)*	(12)	(12)	
Neutral (MPN)	13	11	12	
	(12)	(12)	(12)	
Restrictive (MPR)	12	16	8	
	(12)	(12)	(12)	

*The numbers in parentheses represent the theoretical distribution.

This result is surprising to those who would expect to find a coordination of monetary and fiscal policies. If economic conditions required stimulus they would expect both monetary and fiscal policy to be expansive—and conversely during periods when restriction is appropriate. Table IV suggests precisely the opposite behavior. Subtracting actual from theoretical observations generates a set of positive values (not significantly different from zero) along the northwest—southeast diagonal and a large negative value in the northeast corner. That is, monetary policy has tended to offset fiscal policy.

If we were apologists for policy makers we could—with straight faces—justify this result. We would argue that fiscal policy is set correctly from time to time, while monetary policy is used to "fine-tune" the economy. During such periods the Fed would "lean" against fiscal policy to moderate its impact. At other times, fiscal policy might be unresponsive to the needs of the economy, forcing the monetary authority to carry the entire burden of appropriate counter cyclical policy. 5

The Additive Model

We turn now to efforts to estimate the interaction between monetary and fiscal policy. First we estimate the additive model, equation (1), with the following results:

$$\frac{a_0}{R^2}$$
 $\frac{a_1}{1}$ $\frac{a_2}{1}$ $\frac{a_3}{1}$ $\frac{a_4}{1}$ $\frac{a_5}{1}$ $\frac{$

The values in parentheses are t-statistics; $\overline{\mathbb{R}}^2$, DW, and SE are adjusted \mathbb{R}^2 , Durbin-Watson statistic and standard error of the estimate respectively. These estimates result from the application of the Cochran-Orcutt transformation to reduce the first order autocorrelation in the residuals.

The estimated coefficients produce the combinations of policy impacts shown in Table V. For example, expansive monetary policy coupled with expansive fiscal policy produce a 10.76 percent annual rate of growth in GNP. A gross movement from expansive to restrictive fiscal policy would result in a 0.85 percentage point decline in growth of GNP, calculated by subtracting any element in the left column from its counterpart in the right column. Recall that the additive model makes fiscal policy independent of the mode of monetary policy. But the impact of shifts in fiscal policy is inconsistent in its application. While the movement from expansive to neutral fiscal policy results in a 1.66 percentage point decline in GNP growth, the movement from neutral to restrictive fiscal policy results in a slight rise in GNP growth. This is an unexpected result, and is based on the coefficient of MPR (a₄) that is not significantly different from zero.

Table V

PCLICY CCMBINATIONS AND AGGREGATE DEMAND ADDITIVE MODEL (Equation 1)

Monetary	Fiscal Policy			
<u>Policy</u>	Expansive (FPE) %	Neutral (FPN)	Restrictive (FPR)	
	· ·	~	76	
Expansive (MPE)	10.76*	9 <u>.1</u> 0*	9.91	
Neutral (MPN)	7.84*	6.18*	6.99	
Restrictive (MPR)	5.23*	3.58*	4.39	

*All underlying coefficients are significant at the 0.05 level.

In striking contrast, the impact of monetary policy is greater than that of fiscal policy, and is consistent in its mode-to-mode application. The movement from restrictive to expansive monetary policy results in a 5.53 percentage point increase in the growth of GNP. Moreover, as indicated, the mode-to-mode changes are consistent. A change in monetary policy from restrictive to neutral is accompanied by a 2.60 percentage point increase in GNP growth; movement from neutral to expansive is accompanied by an increase of 2.92 percentage points. These increases do not depend on the fiscal policy mode.

The Interaction Model

We turn now to a discussion of results with the interaction model. Estimates of equation (2) are:

$$\frac{b_0}{2}$$
 $\frac{b_1}{2}$ $\frac{b_2}{2}$ $\frac{b_3}{2}$ $\frac{b_4}{2}$ $\frac{b_5}{2}$ $\frac{b_$

$$\overline{R}^2 = .66$$
 DW = 1.93 SE = 0.64

The adjusted R² of equation (2) is not significantly different from that of equation (1). In equation (1) 5 of the 6 estimated coefficients were significant; in equation (2) 8 of the 10 coefficients are significant. In terms of the Durbin-Watson statistic and the standard error of the estimate it appears that neither estimated equation is superior to the other.

The coefficient estimates of equation (2) produce the combinations of policy impacts shown in Table VI. 8 As in the case of the additive model, the pattern of estimates associated with monetary policy variables is consistent with expectations. That is, regardless of the fiscal policy mode, a movement toward monetary restriction is associated with a reduced pace of GNP expansion. And as before, the pattern of coefficients associated with fical policy changes is not consistent with expectations.

Table VI

POLICY COMBINATIONS AND AGGREGATE DEMAND
INTERACTION MODEL (Equation 2)

Monetary Policy	Fiscal Policy Expansive (FPE) Neutral (FPN) Restrictive (F		
Expansive (MPE)	10.51	8.72	10.42*
Neutral (MPN)	7.33*	6.94*	6.87*
Restrictive (MPR)	6.07*	3.23*	3.59*

*Coefficient is significant at 0.05 level.

In all cases movement from expansive to netural fiscal policy resulted in a reduction of GNP growth, but in two of these cases movement from neutral to restrictive fiscal policy was accompanied by GNP expansion. Moreover, all GNP changes associated with any shift in monetary policy were much greater than any GNP changes associated with a shift in fiscal policy. 9

We now turn to the central question posed in this paper. Does allowance for the interaction of monetary and fiscal policy offer explanatory power not provided in a model based on simple addition of effects? A casual look at the statistics for our regressions and the values shown for the combined effects of monetary and fiscal policies in Tables V and VI suggests a hearty "no" to that question. In only one case was the direction of GNP change associated with mode to mode change in policy different in the two modes. A move from money neutral and fiscal neutral mode to money neutral and fiscal restrictive mode in table VI registers a reduction in CNP growth as expected (but not significant). The same modal move in Table V is accompanied by an increase in GNP growth, contrary to expectations. In all other cases, whether along rows or columns, or along any diagonal, the two models generate the same directional impact of combined monetary and fiscal policies on GNP.

Having argued that there is no essential difference in the two models, we feel it appropriate to point out those results where differences do appear. If we rely on the interactive model—Table VI—monetary changes appear to have greater impact when fiscal policy is neutral or restrictive than when fiscal policy is expansive. That is, movements along columns two and three generate exaggerated changes in GNP growth compared to movements along column one. Recall that in the additive model, changes in GNP accompanying policy changes are the same in all columns (or rows).

Having responded to the central question, we now turn to an assessment of the relative impact of monetary and fiscal policies.

Indeed, the findings presented in both Tables V and VI fairly scream for recognition on this point. Monetary policy has the expected impact on GNP growth irrespective of the mode of fiscal policy. That impact is consistent and marked in every case.

To the contrary, the effect of fiscal policy is mixed, in the expected direction at times, perverse at other times. Beyond the lack of consistency, the effect of changes in fiscal policy are clearly less marked than changes in monetary policy. We take this to be but one more link in the chain of evidence mounting to support the claim that only money matters. Our only finding that offers significant evidence to the contrary comes from the third row of Table VI. Here it is seen that movement from a restrictive to expansive fiscal policy in the interactive model provides a significant boost to GNP growth when monetary policy is in the restrictive mode.

We offer one final scenario that talks to the "crowding out" hypothesis. Suppose we start in the southeast corner of Table VI; monetary and fiscal policies are tight within the context of an interactive model. If we moved to expansive monetary policy, holding to the restrictive fiscal stance, CNP growth would increase 6.83 percent (10.42 - 3.59). If we assume no increase in the rate of government spending or taxes—consistent with the continuing stance of restrictive fiscal policy—the increase in GNP growth would be generated by the private sector.

Consider now a simultaneous change in fiscal and monetary policies to expansion on both fronts, a movement to the northwest corner of Table VI. The resultant increase in CNP growth would be 6.92, hardly different from the 6.83 percent realized with no fiscal expansion. If we assumed the fiscal expansion to have come from increases in government spending, then some part of the increase in CNP growth came from government activity. Since the overall increase was the same as that generated by a change in money alone, the effect of fiscal expansion must have been to "crowd out" private activity.

Summary of Findings

The novelty of our approach is the simplicity of its statistical design. There is complete freedom in coefficient estimation. Hence, any pattern of effects, or interaction, could have emerged. That a similar pattern resulted from both the additive and interaction models, lends credibility to these results.

With respect to the question that motivated this work, the results indicate that a model that allows for interactions between monetary and fiscal policy fails to explain spending better than a simpler additive model. Statistics of significance are essentially the same in both models; and predicted results of an interacting monetary and fiscal policy differ little from predictions emanating from a simple addition of effects.

Another aspect of these results merits special mention. Changes in money appear to matter a good bit more than changes in the government's fiscal position. Monetary expansion in our model was uniformly accompanied by income expansion. These results were generated by the additive

as well as the interactive model; and the effect of money was independent of the stance of fiscal policy.

In striking contrast, the effects of fiscal policy--measured by changes in the full employment surplus or deficit--were mixed. Sometimes the increase in the deficit was coincident with income expansion; at other times the reverse was true. We take this to be but another link in the chain of findings that supports the position that money determines income--leaving fiscal actions to affect the deviation between private and government investment.

M/E/122

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FOCTNOTES

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¹For earlier work see Smyth (1963), Hansen (1973), and Meyer and Hart (1975).

²There was extensive experimentation with alternative measures of monetary and fiscal policies. Results with the monetary base were not materially different from those with the money supply. However, alternative fiscal policy measures produced markedly different results. We have reported the results that do least violence to accepted theory.

This regression gave the following:

% GDI =
$$-0.868 + 2.803$$
 % MI + 0.170 AHED $\overline{R}^2 = .152$ (-0.96) (3.82) (2.41) DW = 1.88

where the numbers in parentheses are t-statistics.

⁵However, the evidence undermines this explanation. Over the 108 quarters monetary policy switched into a new category (e.g., from restrictive to neutral) 42 times. Fiscal policy had 65 such switches, indicating that the thrust of fiscal policy changed more frequently than monetary policy.

⁶For the values in Table V, PD (private demand) is set to its mean value of (approximately) zero.

⁷This result is obtained from $(a_0 + a_1 + a_3) \times 4$. The multiplication by 4 converts the quarter-to-quarter changes to an annual rate.

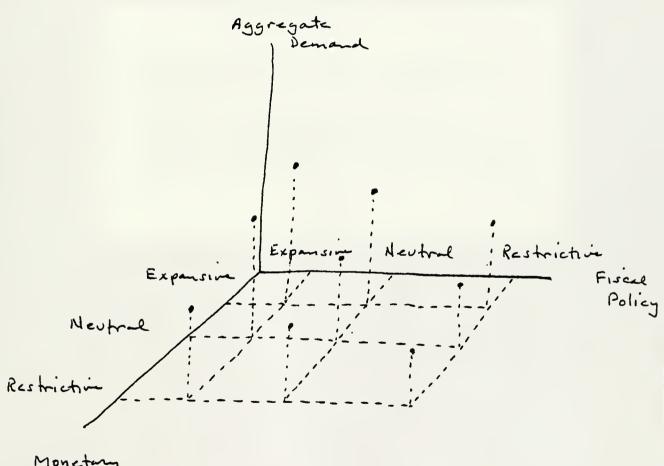
Eased on a chi-square test.

⁸Again, PD was set to its mean value of (approximately) zero.

⁹A t-test for significant differences among the estimates along the rows of Table VI found that the only estimates that are significantly different (at the 0.05 level) are the 6.07 and the 3.23 in the restrictive monetary policy row. To the contrary, tests for significant differences moving down the columns of Table VI found only two instances of estimates that are not significantly different. These two instances are 7.33 and 6.07 in the expansive fiscal policy column and 8.72 and 6.94 in the neutral fiscal policy column.

Figure 1

Aggregate Demand with Alternative Combinations of Fiscal and Monetary Policy



Monetary



Faculty Working Papers

THE MARKETING OF SERVICES; APPLICATIONS IN THE URBAN REALM

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and State University

#665

College of Commerce and Business Administration University of Illinois at Urbana-Champaign Notes

We required the firms to be listed during the entire sample period. The Center for Security Price Research (CRSP) monthly tape was used to select NYSE listed firms. A firm was considered listed if it had monthly stock returns available for the entire sample period.

The absolute percentage error is computed as the average of $\left|\frac{\text{Actual EPS} - \text{Predicted EPS}}{\text{Actual EPS}}\right|.$ Since this error metric can be explosive when the denominator approaches zero we truncated errors in excess of ten to a value of ten. This operation was done for a very small percentage of the cases.







